DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

144. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

A hired a house for one year for \$300; at the end of four months he takes in M as a partner; and at the end of eight months he takes in P. At the end of the year what rent must each pay? [From Greenleaf's *National Arithmetic*, page 442.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.; and J. W. DAPPERT, C. E., Taylorville, Ill.

1st Method. \$300 for one year is \$25 per month. A has the house alone for four months at \$25=\$100. He shares it with M for four months at \$12½ each =\$50. With M and P for four months at \$8⅓ each=\$33⅓.

∴ A pays 100+50+33⅓=\$183⅓.

M pays 50+33⅓=\$83⅓.

P pays \$33⅓. DAPPERT.

2nd Method. A, 12 months+M, 8 months+P, 4 months=24 months for one person.

\$300÷24=\$12½ per month.

12×12½=\$150, what A pays.

8×12½=\$100, what M pays.

4×12½=\$50, what P pays.

By the second method the house rents for \$12½ per month for four months, \$25 per month for four months, and \$37½ per month for four months.

The modern idea would call the first method the correct one.

Also solved by *W. P. WEBBER*, and *H. C. WHITAKER*.

NOTE.—Problems of this nature have been the subject of much discussion in the past. The first method is unquestionably the correct one, as a person is required to pay rent for the time he occupies the house. Coach problems, where A hires a coach for a certain sum and then on the way takes in B, and then a little farther on takes in C, what amount should each pay? should be solved on the basis that each pays in proportion to the distance he rides, such problems being of the same nature as the one under discussion. ED. F.]

145. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield. Mo.

By discounting a note at 20% per annum, I get 22½% per annum interest; how long does the note run? [From Ray's *Higher Arithmetic*, page 405.]

Solution by S. F. NORRIS, Professor of Astronomy and Mathematics, Baltimore City College, Baltimore, Md., and W. P. WEBBER, Houston, Miss.

Face × Rate × Time (in years)=Discount....(1).

Proceeds × rate × Time (in years)=Interest....(2).

In this case, the discount equals the interest; hence, $P\times r\times T=F\times R\times T$.

Cancelling T and substituting the two rates, $P\times\frac{9}{40}=F\times\frac{1}{5}$.

$\therefore\ P=\frac{8}{9}F.$

Assuming \$100 for the face of the note, the proceeds will be \$88$\frac{8}{9}$, and the discount or interest \$11$\frac{1}{9}$.

From (2), $\text{Time}=\dfrac{\text{Int.}}{P\times r}=\dfrac{11\frac{1}{9}}{88\frac{8}{9}\times\frac{9}{40}}=\frac{5}{9}$ year=200 days.

Also solved by *G. B. M. ZERR*, and *H. C. WHITAKER*.

ALGEBRA.

119. Proposed by HARRY S. VANDIVER, Bala, Montgomery County, Pa.

Given $\tan x=x+\dfrac{x^3}{3}+\dfrac{2x^5}{3\times5}+\dfrac{17x^7}{3^2\times5\times7}+\dfrac{62x^9}{3^2\times5\times7\times9}\cdots\cdots$

Find the general term and interval of convergence of this series.

I. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Using the notation of the Calculus, let $\tan x$ be represented by $f(x)$ or simply f. Then dy/dx will be $f'(x)$ or more simply f; similarly, f'', f''', f^{iv}, f^{v}, etc., will be used. We have then by differentiating :

$f(x)=\tan x$ $\qquad f(0)=0.$

$f'=\sec^2x$ $\qquad f'(0)=1.$

$f''=2\sec^2x\tan x=2ff'$ $\qquad f''(0)=0.$

$f'''=2ff''+2f'^2$ $\qquad f'''(0)=2.$

$f^{iv}=6f'f''+2ff'''$ $\qquad f^{iv}(0)=0.$

$f^{v}=6f''^2+8f'f'''+2ff^{iv}$ $\qquad f^{v}(0)=16.$

$f^{vi}=20f''f'''+10f'f^{iv}+2ff^{v}$ $\qquad f^{vi}(0)=0.$

$f^{vii}=20f''+30f''f^{iv}+12f'f^{v}+2ff^{vi}$ $\qquad f^{vii}(0)=272.$

We see that the *even* differential coefficients vanish and the *odd* follow a remarkable law : The first term is the middle term in the expansion by the Binomial Theorem and the following terms are the double of the successive coefficients of the Binomial Theorem.

By this law we can write any of the odd derivatives, thus :

$f^{ix}=70f^{iv2}+112f'''f^{v}+56f''f^{vi}+16f'f^{vii}+2ff^{viii}$ $\qquad f^{ix}(0)=7936.$

$f^{xi}=252f^{v2}+420f^{iv}f^{vi}+240f'''f^{vii}+90f''f^{viii}+20f'f+2^{ix}ff^{x}$ $\qquad f^{xi}(0)=353792.$

$f^{xiii}=924f^{vi2}+1584f^{v}f^{vii}+990f^{iv}f^{viii}+440f'''f^{ix}+132f''f^{x}+24f'f^{xi}+2ff^{xii}$

whence $f^{xiii}(0)=22368256$, and thus indefinitely.

That is, in writing f^{xiii}, I write the coefficients of $(a+b)^{12}$, which are 1, 12, 66, 220, 495, 792, 924, etc. I set down 924 and double each of the others and get the result as given above.